

Infinite Virtual Stoa

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1. Introduction

Stoicism is a philosophy that considers the object of life to be ataraxia (αταραξία), a state of psychological stability which is undisturbed by exposure to phenomena and circumstances that lie outside one's control. Such circumstances may include ill health, poverty, natural disasters, corrupt social orders and unrequited love, and may cause loss of composure and mental balance through feelings of pain, humiliation, insufficiency, envy or greed. Stoicism is a coherent system of powerful ideas about how to pursue a life of equanimity in the face of adversity which has nurtured philosophy and psychology to this day. The founders of Cognitive Behavioural Therapy have cited Stoicism as their main inspiration [1]. Stoicism flourished in ancient Athens and Rome at a time when ancient democracy was dying and people experienced loss of control over their lives under authoritarian and imperial regimes. In an age of serious global economic, environmental and psychological uncertainty and crisis, Stoicism has still pressing and valuable lessons to teach us about calm, composure, resilience and emotional stability.

Stoicism owes its name to Stoa Poikile ('painted porch' in Greek), a colonnaded building in the Athenian agora where Zeno of Citium founded his school in the 4th century BCE. In this project we develop an 'Infinite Virtual Stoa' to host an expandable online repository of resources about Stoicism. Our repository exists in the space defined by the Stoa, which is a colonnaded building that takes the form of the ancient, and sacred to many cultures, geometrical motif known as 'the flower of life'. The building is composed of multiple evenly-spaced, overlapping circles arranged in a flower-like pattern with six-fold symmetry that is potentially expandable to infinity. Our Stoa is immersed in water, with pools regularly forming in the space between the colonnaded walks that delineate the space and becomes an art gallery-library for the exhibition of online stoic resources: quotes, stories, books, paintings and videos.



Aerial view of the infinite Stoa

Stoicism explored patterns of psychological fallacy on one hand, and patterns of thought and behaviour for the pursuit of ataraxia on the other. Our Stoa is built on a geometrical pattern that is expandable to infinity. Thus the concept of pattern is central to this paper. Firstly, we focus on patterns as they occurs in Mathematics, Science, the Arts and the mind. Then we discuss patterns within Stoicism and the design of the infinite Stoa. Finally, we highlight the long term goals of this project and its further evolution.

2. Téchnē and ‘Arthematics’

We start from a vision where Mathematics, Geometry, and Computer Science may be embraced in a sense of the unified arts and technology suggested by the ancient meaning of the Greek word téchnē (τέχνη) [2]. Stefanie Mandelbaum interestingly coins ‘Arthematics’ as a crossing field between the Arts and Mathematics [3]. This goes also in line with another neologism, that of ‘Artification’ [4], which gives name and meaning to the fusion of the artistic and the - apparently - non-artistic, that then sublimates to a new Art. In this context, we are specifically interested in the dynamics between Artification of Games and Gamification of Arts.

2.1 Mathematics, patterns and the Arts

The appearance of perspective in Renaissance painting is intimately related to the influence of ‘the science of sight’ [5] and the mathematical notion of infinity. It is an example of mathematical thinking that proved to be instrumental in the making of artistic masterpieces. This advance in the Arts was driven by the Renaissance artists’ brave geometrical thinking including the contemplation of insightful but counterintuitive ideas, such as equating the existence of parallel lines that do end up meeting [6].

Deeper in history, humanity has left us with an ancient panoply of beautiful patterns still exposed in old walls, windows, floors and ceilings, or visible in other kinds of artefacts - such as ceramics - of antique cultures like the Assyrian, Greek, Roman, Byzantine, Iranian and Arabic [7]. This heritage has inspired artists through times. Technically, the motifs and regularities can be classified by periods and styles or even in more systematic ways and they too have an intimate connection with mathematical knowledge. These ancient arrangements are still present in contemporary works of the decorative arts of ornament and in prints. Yet their influence is not limited to those fields of creativity. Escher, for instance, has used them - and has also developed his own ideas over them - in his engravings and drawings introducing the visual senses into new heights of thinking geometrically.

Perhaps the harmonies of symmetry, isometry and geometry are desirable and explain that the order present in the universe is also present in cultural artefacts such as Art, Music, and Architecture [8]. Such thoughts led Ackerman to state that 'the modern argument of form versus structure is as meaningless as the mediaeval argument of *ars* versus *scientia*, for it likewise disrupts a partnership which can function only in happy union' [9].

In Music, as in the visual Arts, Geometry and Techne also coexist. In musical composition, geometric transformations and symmetry are known to have played a role in the works of Bach and of Bela Bartok [10]. There are also known composition symmetries such as the 'Friezes patterns' [10] and tonal and rhythmic patterns, such as dissonance, two-voice vs. coloration, and ostinato patterns [11]. These are resourceful tools for the composer because patterns in music can also be derived from one into another by variation [12].

2.2 Historic technologies in the service to the Arts

The history of the synthesis of Mathematics and the Arts is also exemplified by several engineering artefacts such as the tools that have helped artists to achieve extraordinary paintings and sculptures. From the chisel to calligraphy pens, the compasses and the proportional dividers, to more complex tools for achieving a correct perspective in painting, such as the perspective machine depicted in Leonardo's 'Draftsman drawing an armillary sphere' [13] or the pantograph, a linkage device invented in 1630 by Christoph Scheiner that makes it possible to draw a scaled copy of a smaller depiction [14]. The automation that nowadays is the focus of interest of the Generative Art is also rooted in history, from the legendary Antikythera mechanism to the pervasive echoes of the music of the automatic carillons of Mafra [15] and the writing, drawing and musical automata of Jaquet-Droz [16].

3. Patterns and Computer Science

Patterns are also prevalent in Computer Science, and that indeed is the case in the fields of pattern recognition and machine learning [17]. Pattern recognition searches for regularities in data and deals with their automatic classification [17]. In Computer Science, a pattern is an abstraction of an object that can be understood as a class described by certain attributes and that can be searched for in the data [18]. Patterns

can usually be recognized statistically or syntactically [18]. It is, for instance, possible to run pattern matching algorithms in a musical piece to study its 'motifs and their variants' [12].

3.1 Repetition, tilings and tessellations in Computer Graphics

Going back to the works of Leonardo da Vinci and the Renaissance, it is remarkable to see that his description of dealing with perspective essentially matches the contemporary approach for rendering a 3D scene to a computer screen. Long before the Renaissance, the Romans had also used small tiles – tesserae - to build mosaics that could fill the plane with depictions. This regular tiling of the plane or even space is hence called Tessellation. Still today, the 'tessellation stage' is one of the steps of current graphics 3D rendering pipelines. Other kinds of regularities are also thought fundamental in Computer Graphics. The symmetrical divisions of the plane and of higher dimensions are crucial for rendering pictures, movies and geometries as they can be reduced to structural repetitions of bidimensional pixels and three-dimensional voxels.

3.2 Pattern generation

Efforts are also directed towards understanding pattern formation and to 'describe universal classes of pattern' in a 'precise formalism which serves as the conceptual basis for synthesizing and analysing patterns' [19].

3.3 Proceduralism, and digital games

Proceduralism in the realm of digital games allows the automatic creation of infinite and randomised worlds. These can be populated with procedural models and game objects, geometries, textures, levels, AI behaviours, and with impact on the dynamics of narratives, story, and in music generation where, for instance, musical piece sections are repeated according to dynamic patterns [20]. Dynamic music can be parametrised by variations of tonality, rhythm, harmony, 'andamento', pitch, and themes and can have an impact both in immersion and inclusively play with the narrative and player interaction, as in the use of the 'leitmotiv' in dynamic game music [21].

4. Patterns and the mind

There are two noticeable connections between the realm of patterns and the human mind. The first is that the mind has the tendency to be attuned and attracted to regularity as in symmetric patterns, in which it finds beauty [22] [23]. The second is that patterns have an influence in processes and flows within the human mind. For instance, temporal patterns have the power to induce an 'inner clock' that influences the perception of music [24]. One recognises and remembers different birds by the melodies they sing. Studies in psychology have also investigated intrinsic and extrinsic structures of stimuli and their relationship with discrimination, classification, judgement of similarity [25] and, particularly in music, in long-term memorisation of melodies [26]. Additionally, the repetition of static images and sculptures are also

interpreted as 'visual rhythm, or an impression of coherence and movement' [27]. The 'same measure' of patterns is also mentioned in some mental pathologies, as it is the case of the obsession with symmetry in Obsessive Compulsive Disorders [22].

5. Stoic patterns

Stoicism is an ancient philosophy that encompasses logic, physics and ethics in a comprehensive philosophical system from which much can still be learned about tranquillity, resilience, mental balance as well as tolerance and openness in society. Stoic philosophers used the analogy of a 'garden' to describe their system [28]. In this analogy, philosophy is seen as a garden that it is fenced by logic protecting it from erroneous reasoning often motivated by imperfections in human nature. Within the garden, physics is the soil where we cultivate our understanding of the world including that of our human nature. The fertile soil of physics in turn yields the fruits of philosophy which for Stoicism is the 'ethics', or living a 'good life' characterised by serenity and justice that can be experienced individually and collectively.

Stoics have made contributions in many areas of intellectual enquiry. Chrysippus, for example, is known to have founded propositional logic, an early form of the formal reasoning system employed in contemporary analytical philosophy and Computer Science [29]. However, one of their most significant and lasting contributions is in the area of psychology. Stoicism is indeed the first systematic attempt to understand patterns of fallacious reasoning that cause disturbances in human psychology and behaviour; and stoics proposed remedies which still inspire modern cognitive therapy [1]. For the purposes of this paper, and its discussion of patterns, we focus on three areas of human behaviour where stoics identified dysfunctionalities caused by fallacious thinking and where they proposed interesting remedies: anxiety, agitation and tribalism.

5.1 Anxiety

Many of us tend to live in constant anxious anticipation of stressful events, e.g. fearing loss of status, wealth, health, or reputation. According to the stoics such anxieties are largely caused by mental confusion and inability to consider a basic dichotomy between those things that we can control and those that we cannot. Status, wealth, and health are largely defined by external events which can be random and outside one's control. Realising this can lead to a fundamental shift in how one then responds to unfortunate events. It is not the event itself that matters - argue the stoics - but our responses to it, and these can be adjusted. Once one understands, for example, that a job promotion is not entirely in one's control, but depends on circumstances and personalities of others, then failure to obtain the promotion becomes less personal and hurtful and therefore one can be less anxious about the outcome and rather focus on the effort. Stoics instead propose that life should prioritise the pursuit of four cardinal virtues: wisdom, courage, justice and temperance, and all these are entirely in our control. A good life is defined as acting with wisdom, be courageous and doing the right thing, treating others justly and responding to events with moderation. Status, wealth, health and reputation, on the other hand, are simply defined as 'preferred indifferents' i.e. things that may be nice

to have but one should easily part with [30]. Stoics developed some excellent advice around this theme. Epictetus, one of the eminent stoics, for example, advised us to think of our loved ones not as possessions but as borrowed from the universe [31]. When the time comes to return them, we should not be devastated but be grateful for the time we had with them as we would be grateful when we have returned a good book lent by a friend.

3.2 Agitation and anger

Stoicism attributes agitation and anger to overt optimism and unrealistic expectations. In his *Enchiridion*, Epictetus gives a lucid illustration of this using the example of a citizen visiting the Roman baths for cleansing, relaxation and recuperation.

"If you are going to bathe, picture to yourself the things which usually happen in the bath: some people splash the water, some push, some use abusive language, and others steal. Thus, you will more safely go about this action if you say to yourself, 'I will now go bathe, and keep my own mind in a state conformable to nature. For thus, if any hindrance arises in bathing, you will have it ready to say, 'It was not only to bathe that I desired, but to keep my mind in a state conformable to nature; and I will not keep it if I am bothered at things that happen' " [32].

For the stoics 'nature' in the above effectively means 'reason' (logos), because nature is both logical in its regularities and has gifted humanity with the capacity to reason. Epictetus makes a very apt point: overt optimism disappoints and agitates. Indeed, many of us regularly get upset about traffic jams and aggressive drivers as if we lived in a world where these things were extinct. We use computers for online banking but hope never to suffer malware, and when we do we get angry. However, Stoics had a very clear view of anger as temporary suspension of mental faculties. In his famous essay, Seneca described anger as temporary madness [33]. The stoic message is that unrealistic expectations lead to disappointment, agitation and anger. Thus, it is preferable to be realistic, and when we encounter potential triggers of anger, create mental space between external events and our responses to allow reasoning and calm reflection to take place.

3.3 Tribalism

Tribalism assumes that a strong cultural or ethnic identity, often based on proximity or kinship, separates one member of a group from the members of another group. Various forms of tribalism, including nationalism, have historically exploited false and simplistic conceptions of history which glorify the tribe or the nation, and exclude other groups.

The stoics developed an antithesis to tribalism based on a cosmopolitan view of human nature and society. According to the cosmopolitan view, we are social animals who participate not in one but many groups which are increasingly larger. In each of those groups we may assume different roles which, if performed well, lead to collective benefit. The word cosmopolitan derives from the Greek κοσμοπολίτης

meaning being 'citizen of the world'. Hierocles, an eminent Stoic, developed a brilliant illustration of the concept by placing the individual in the centre of a series of expanding circles of concern: the smallest circle is the individual itself, next comes the close family, the extended family, the city, the neighbouring cities, the country, and finally the larger circle is humanity. Our task - stated Hierocles - is to compress those circles so as to bring humanity closer to the core of our concerns [34]. Cosmopolitanism thus becomes the idea that all humans belong to one community based on our shared nature and capacity for logic and morality.

6. The infinite Stoa

Stoicism has lasted millennia and still carries valuable lessons for humanity. There are contemporary communities of practising stoics, hence there is no reason to believe that stoicism will cease to exist and develop in the future. To facilitate this, we propose an infinite virtual Stoa that will enable hosting an expandable online repository of resources about Stoicism. The internet is already potentially infinite, but we propose to create a more interesting virtual space that offers artistic possibilities and sensibilities for the presentation and exploration of stoic ideas. Thus, we configure our gallery as a virtual colonnaded Stoa that is reminiscent of the original Stoa Poikile. Our Stoa has no boundaries, being allowed to grow *ad infinitum*. Below we explain how this is achieved.

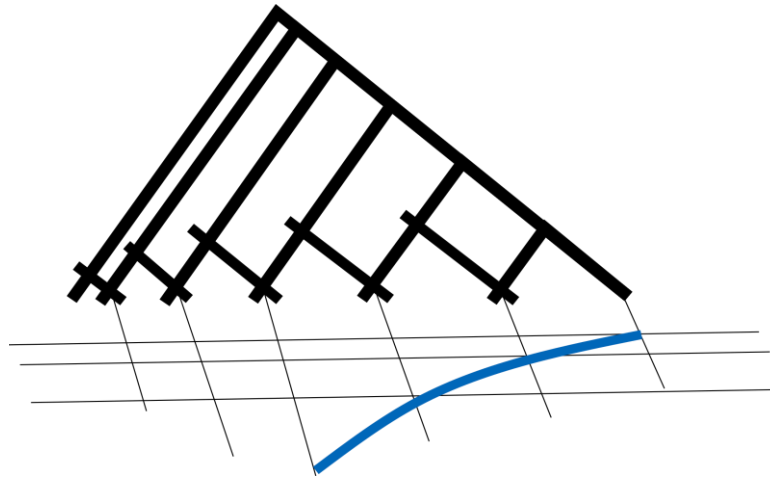
6.1 The geometry of the circle

Computationally, an infinite virtual Stoa can be built procedurally. This requires some form of structural arrangement that can be repeated in the Stoa's architecture. We could have created a linear Stoa but have chosen a more interesting motif of seven overlapping circles known as the 'flower of life'. This arrangement has the particularity that by the connection of all points another pattern can be found, one that combines the projections of all platonic solids, known as the 'Metatron Cube', that is a figure that shows 'perfect proportions and relations between its geometric components' [35]. This hidden pattern is also known since antiquity and archaeologists account finding it in the architecture of a defensive structure of the Copper Age in the Iberian Peninsula [35].

6.2 Pantographs and Splines

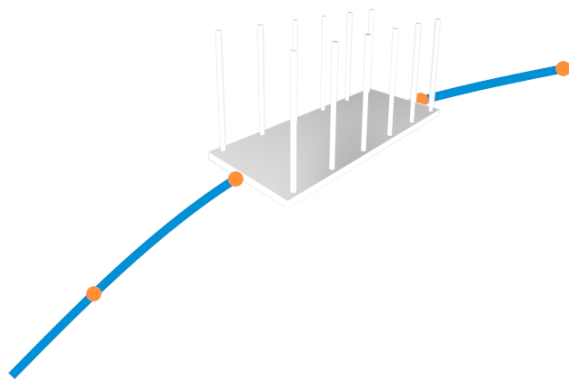
The prominent technical device necessary for building the pattern is the spline, a computational resource with roots in the ancient art of making tools [36], specifically the pantographs that have been conceived and used for drawing elliptical arches in the 17th century [37].

In the 1950s, automotive design required novel ways for the representation of volumes' and shapes' which led to the invention of computational splines [38]. The invention of splines moved from the elliptic pantograph to defining curves algorithmically, an innovation that also moved on from defining the arch inside a sectioned 2D rectangle, to a 3D parallelepiped [38].



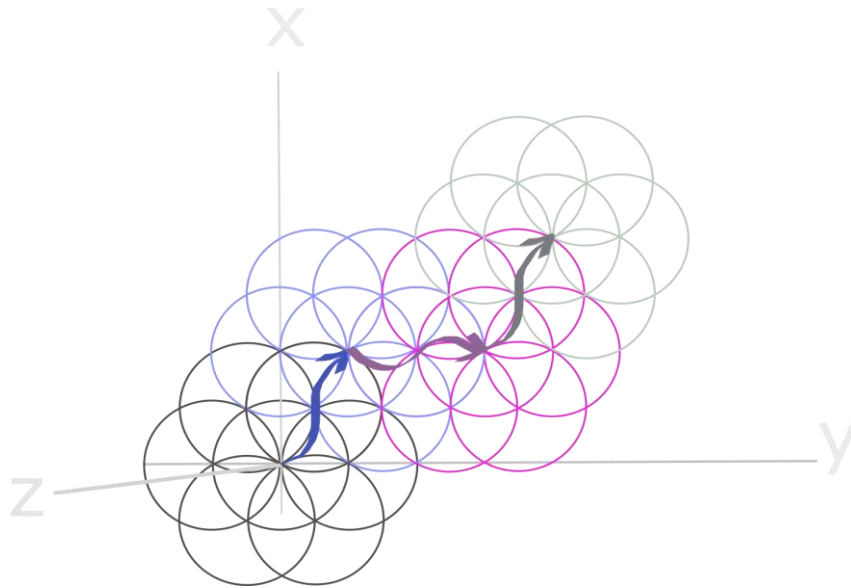
Drawing ellipses before algorithms, with a pantograph (diagram based on [38])

Splines are nowadays present in all kinds of drawing and modelling software, and also present in digital game engines, along with multitudinous other technical advances in the service to the Arts, that mostly have all had their roots in historical engineering. Among the contemporary applications, splines can, for instance, be used for extracting the geometrical features on digitally reconstructed parametric model representations of objects [39].



The Stoa's modelled section is reproduced along a spline

By resourcing to splines, it is possible to section them equally and populate their parts with 3D models. In the Stoa, the spline sections are associated with a reduced 3D model of a small motif of a segment of a Stoa, that can be repeated, and then sections are harmoniously collated. Every spline instantiates a circle of the flowered pattern and infinity is achieved by making the rendering of the Stoa dependent on the player's navigational directions. The circles also have anchor points that can work as the origins for further splines/circles. So, by being generated on the fly, in conformity with the navigation of the viewer, the Stoa becomes virtually infinite in any direction of browsing. The result is that moving towards the horizon creates new space. This is a characteristic of what is known in game development as 'procedural world generation'. Whilst, by navigating, the world is created instantly, it is also possible to recreate previous paths towards where the audio-visual sculptures have been situated.



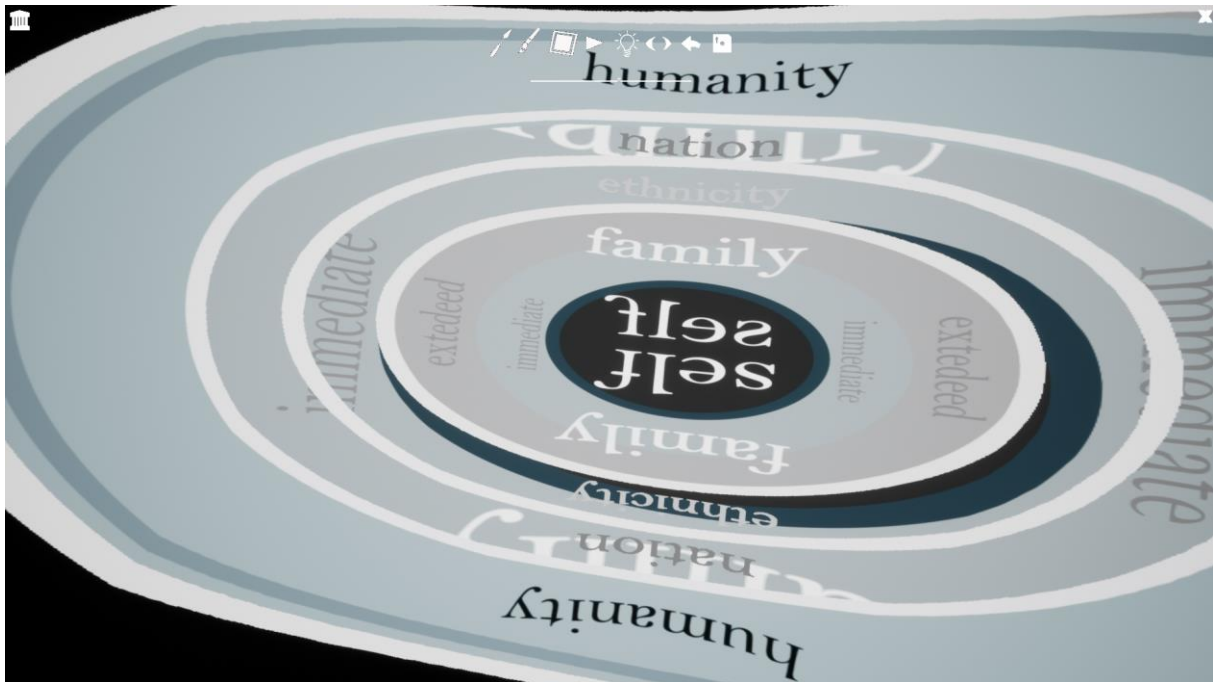
Process of the splines being generated during navigation, viewed from above

6.3 A procedural art gallery for viewing artworks

Navigating the Stoa resembles walking on an infinite unbounded labyrinth. Along the continuous paths, the player can find the audio-visual sculptures that can be designed with the resources provided by TIMAEUS. TIMAEUS is a virtual art studio, which has been inspired by Plato's homonymous work, especially in its narrative about the Platonic realm of geometrical thinking. The sculptures contain audios, films, pictures and texts that are continuously being displayed and that can be visited from inside by the wanderer. An example sculpture inspired by Hierocles' circles of concern is shown below.



Hierocles-themed sculpture viewed from the Stoa



The sculpture viewed from the inside

7. Conclusion and future

The world seems to be full of recurring patterns that repeat themselves in completely different areas. The branching of the trees is similar to that of veins, the delta of rivers, the wrinkles of a palm and cracks in dry soil. In this paper we discussed the concept of pattern as it occurs in Mathematics, Science, the Arts, the mind, and the philosophy of stoicism. Stoicism is a philosophy that explored patterns of psychological fallacy on one hand, and patterns of thought and behaviour for the pursuit of ataraxia on the other.

Building on the themes of patterns and Stoicism, we designed an infinite virtual Stoa using a classical geometrical pattern that is expandable to infinity. The Stoa could be used to host and present in an artistic manner an infinitude of past and future resources about Stoicism. It can potentially host areas of study devoted to great philosophers, like Epictetus, Seneca and Marcus Aurelius, or thematic areas where stoic ideas are explained with quotes, text, animations and videos. Specific ideas or themes can be presented in a creative fashion to the viewer as multimedia illuminated sculptures using TIMAEUS. The gallery is designed to be accessible online.

Finally, this Stoa can be seen as a serious game for education and learning or a development towards Gamification of the Arts, a topic that we will further explore in the future.

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